

Exact travelling wave solutions of time-fractional Clannish Random Walker's Parabolic equation and sensitive analysis

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Abstract

In this article, the new extended direct algebraic method is used to build the exact travelling wave solutions for the non-linear time-fractional Clannish Random Walker's Parabolic equation. This study establishes the framework in which periodic, singular, dark, bright and kink-type wave solitons are obtained with exact solutions offered by the mixed hyperbolic and trigonometry solutions, mixed periodic and periodic solutions, plane solutions, shock solutions, mixed trigonometric solutions, mixed singular solutions, mixed shock single solutions, complex solitary singular solutions, shock solutions and shock wave solutions. The exact soliton solutions of the time-fractional clannish random walker's parabolic equation model is graphically presented at different values of involved parameters by **Wolfram Mathematica** software. The propagating behaviours display in 3-D, contour and 2-D surfaces of the obtained results to visualise the impact of the involved parameters. The sensitivity analysis is demonstrated for the wave profiles of the designed dynamical framed structure, where the soliton wave number parameter regulates the water wave singularity. This analysis illustrates that the utilized method is efficient and reliable and can be useful to find appropriate solutions in closed-form solitary soliton to a wide range of non-linear evolution equations. These techniques can also be utilised to find new exact travelling wave solutions for any class of integer or fractional differential equations that arise in mathematical physics.

0.1 keywords:

New extended direct algebraic method; Time-fractional Clannish Random Walker's Parabolic equation; Exact solutions; Sensitive analysis.

1 Introduction

The time-fractional Clannish Random Walker's Parabolic (CRWP) equation [1, 2] determine the behaviour of two random walker's species A and B who possess out one-dimensional and concurrent random walk characterised by a rise in the clannishness of members of the one species A at point x at time t , $u(t, x)$ can be written by the time-fractional (CRWP) equation as,

$$D_t^{\alpha_1} U + jU_x + kUU_x + lU_{xx} = 0 \quad (1)$$

where α_1 denotes the order of the fractional time derivatives and $0 < \alpha_1 \leq 1$.

Exact solutions for travelling waves for the non-linear fractional partial differential equations (FPDEs) are important in the study of physical phenomena. In recent years, fractional equations, both partial and ordinary, have been used in the modelling of many chemistry, engineering, physics, and biology problems [3]. In the literature, several definitions of the fractional derivatives are available, including Riemann Liouville [4, 5], the conformable fractional derivative [6, 7], and new truncated M-fractional derivative [8]. The non-linear (PDEs) are essential for investigating the non-linear physical situation. (PDEs) play an important role in many analyses due to their intermittent appearance, well-designed and potential in various non-linear fields. In order to find the exact solitary wave solution in various aspects, several authors used various approaches [9–13]. The accurate and solitary wave solution of the (PDEs) was obtained by many authors using various approaches in various contexts. In both pure and applied mathematics, non-linear (PDEs) have become more significant and useful in recent years. (PDEs) are important for many technical and physical problems and no one can ignore their importance.

The travelling wave hypothesis has evolved into the primary method for analyzing as well as extracting soliton solutions to the many non-linear evolution equations (NLEEs). Researchers have paid close attention to closed forms of exact solutions in recent years. Such solutions are extremely important in studying the stability and nature of physical systems. The bell and kink-shaped solitons are commonly used to simulate the non-linear oscillatory events in plasma, hydrodynamics and fibre-optics. In recent years, various types of closed-form solutions of (NLEEs), such as soliton, rational, coupon, periodic and quasi-periodic solutions, have been reported. Non-linear PDEs have a very vast range of practical applications in the field of wave theory, including transportation of heat and mass, plasma physics, and hydrodynamics chemical technology [14, 15], ocean waves process may be characterized by non-linear ordinary differential equation systems [16, 17], population ecology [18], electromagnetic wave interaction in a plasma [19], non-linear may appear in quantum mechanics in several different ways [20, 21] and so on.

The story of solitons begins with John Scott Russell's observation of the translation wave. Before Russell's work was proven in the 1870s, prominent philosophers and scientists widely praised its scientific implications. Nonetheless, Rayleigh and Boussinesq's work demonstrates the critical issue of non-linearity and dispersion. It is still rendering to address Stokes and Airy's argument against using kink-shaped and bell-shaped solutions to simulate wave phenomena in the optical fibre, the elastic media and other important fields, well-known examples include the travelling wave solutions of Korteweg de Vries and the Boussinesq equation [22–25].

There are multiple approaches and schemes such as the sine-Gordon expansion scheme [26], the Kudryashov method [27], the simply extended equation method [28] and the bilinear neural network technique [29], used to obtain exact soliton solutions for non-linear partial differential equations, [30] variational iteration method [31], extended exponential function method [32], Hirota bilinear technique [33], power series method [34], F-expansion technique [35] as well as several others [36–42].

In the literature, there are a few approaches that are commonly used for obtaining exact solutions to the integrable wave equation (1), adapted (G/G) -expansion scheme [43], modified extended auxiliary equation mapping [44] modified F expansion method [45]. Compared to the current analytical technique, the proposed method is more dependable and computationally efficient. The proposed model used various integrating techniques to extract dark, bright and singular solitons.

To our knowledge, the time-fractional (CRWP) equation model has not yet been studied using the new extended direct algebraic method and sensitivity and chaos theory is also ignored. As a result, the new extended direct algebraic method approach is used to find stable soliton solutions to non-linear (FPDE), especially the time-fractional (CRWP) equation. The present methodology has some advantages over the previously studied techniques in the form of a more generalized solution and its performance is more efficient and effective. The solitonic patterns of the time-fractional (CRWP) equation have been illustrated, via exact solutions offered by travelling wave solutions obtained, including periodic and singular waves, bright and dark waves and kink-type wave solutions by using a new extended direct algebraic method. The obtained solutions are represented as the mixed hyperbolic and trigonometry solutions, periodic and mixed periodic solutions, plane solutions, shock solutions, mixed trigonometric solutions, mixed singular solutions, mixed shock single solutions, complex solitary singular solutions, shock solutions and shock wave solutions. Comparisons are provided diagrammatically for generalized time-fractional (CRWP) equation solutions, which are represented graphically in Mathematica by varying the embedded parameter values. The applicability of the obtained typical solitary wave solutions of the considered model has been investigated here by drawing 3-D, 2-D and contour graphs of the obtained solitons. In the end, sensitivity analysis on

the newly designed dynamical structural system for wave profiles is illustrated to study the dynamic behaviour of the considered model. We are hopeful that our findings will help physicists predict new ideas in mathematical physics and its applications.

In section 2, we used the new extended direct algebraic method to develop exact solutions. Section 3, demonstrates the application of the proposed method. Furthermore, we observed different wave appearances in the 3-D, contour and 2-D graphical representations of the soliton solutions for different values of the wave velocity and discussed the graphical presentation of research findings in section 4. Section 5, graphically depicts the sensitivity visualization, as well as the analysis and discussions of the results. Section 6 contains the conclusion of this study.

2 Description of method

2.1 Illustration of proposed technique

Consider the general (NPDE) of the type:

$$X(U, U_t, U_x, U_{tt}, U_{xx}, \dots) = 0. \quad (2)$$

It can be converted into (ODE) which can be obtained as,

$$Y(N, N', N'', \dots) = 0. \quad (3)$$

by means of the following transformation,

$$U(x, t) = N(\zeta), \quad (4)$$

where $\zeta = k_1x + k_2t$ and k_1 and k_2 are real constants and can be changed according to the physical phenomena. Consider the following solution of Eq. (3):

$$N(\zeta) = a_0 + \sum_{i=1}^M [a_i (\mathfrak{F}(\zeta))^i], \quad (5)$$

where,

$$\mathfrak{F}'(\zeta) = \ln(A)(\wp + \mathfrak{B}\mathfrak{F}(\zeta) + \lambda\mathfrak{F}^2(\zeta)), \quad A \neq 0, 1, \quad (6)$$

where, $\wp, \mathfrak{B}$ and λ are real constants and $S = \mathfrak{B}^2 - 4\wp\lambda$. The general solutions with respect to parameters $\wp, \mathfrak{B}$ and λ of Eq. (6) are:

(Case 1): When $\mathfrak{B}^2 - 4\wp\lambda < 0$ and $\lambda \neq 0$,

$$\mathfrak{F}_1(\zeta) = -\frac{\mathfrak{B}}{2\lambda} + \frac{\sqrt{-S}}{2\lambda} \tan_A\left(\frac{\sqrt{-S}}{2}\zeta\right), \quad (7)$$

$$\mathfrak{F}_2(\zeta) = -\frac{\mathfrak{B}}{2\lambda} - \frac{\sqrt{-S}}{2\lambda} \cot_A\left(\frac{\sqrt{-S}}{2}\zeta\right), \quad (8)$$

$$\mathfrak{F}_3(\zeta) = -\frac{\mathfrak{B}}{2\lambda} + \frac{\sqrt{-S}}{2\lambda} (\tan_A(\sqrt{-S}\zeta) \pm \sqrt{uv} \sec_A(\sqrt{-S}\zeta)), \quad (9)$$

$$\mathfrak{F}_4(\zeta) = -\frac{\mathfrak{B}}{2\lambda} + \frac{\sqrt{-S}}{2\lambda} (\cot_A(\sqrt{-S}\zeta) \pm \sqrt{uv} \csc_A(\sqrt{-S}\zeta)), \quad (10)$$

$$\mathfrak{F}_5(\zeta) = -\frac{\mathfrak{B}}{2\lambda} + \frac{\sqrt{-S}}{4\lambda} (\tan_A\left(\frac{\sqrt{-S}}{4}\zeta\right) - \cot_A\left(\frac{\sqrt{-S}}{4}\zeta\right)). \quad (11)$$

(Case 2): When $\mathfrak{B}^2 - 4\wp\lambda > 0$ and $\lambda \neq 0$,

$$\mathfrak{F}_6(\zeta) = -\frac{\mathfrak{B}}{2\lambda} - \frac{\sqrt{S}}{2\lambda} \tanh_A\left(\frac{\sqrt{S}}{2}\zeta\right), \quad (12)$$

$$\mathfrak{F}_7(\zeta) = -\frac{\mathfrak{B}}{2\lambda} - \frac{\sqrt{S}}{2\lambda} \coth_A\left(\frac{\sqrt{S}}{2}\zeta\right), \quad (13)$$

$$\mathfrak{F}_8(\zeta) = -\frac{\mathfrak{B}}{2\lambda} + \frac{\sqrt{S}}{2\lambda} (-\tanh_A(\sqrt{S}\zeta) \pm \iota\sqrt{uv} \operatorname{sech}_A(\sqrt{S}\zeta)), \quad (14)$$

$$\mathfrak{F}_9(\zeta) = -\frac{\mathfrak{B}}{2\lambda} + \frac{\sqrt{S}}{2\lambda} (-\coth_A(\sqrt{S}\zeta) \pm \sqrt{uv} \operatorname{csch}_A(\sqrt{S}\zeta)), \quad (15)$$

$$\mathfrak{F}_{10}(\zeta) = -\frac{\mathfrak{B}}{2\lambda} - \frac{\sqrt{S}}{4\lambda} (\tanh_A\left(\frac{\sqrt{S}}{4}\zeta\right) + \coth_A\left(\frac{\sqrt{S}}{4}\zeta\right)). \quad (16)$$

(Case 3): When $\wp\lambda > 0$ and $\mathfrak{B} = 0$,

$$\mathfrak{F}_{11}(\zeta) = \sqrt{\frac{\wp}{\lambda}} \tan_A(\sqrt{\wp\lambda}\zeta), \quad (17)$$

$$\mathfrak{F}_{12}(\zeta) = -\sqrt{\frac{\wp}{\lambda}} \cot_A(\sqrt{\wp\lambda}\zeta), \quad (18)$$

$$\mathfrak{F}_{13}(\zeta) = \sqrt{\frac{\wp}{\lambda}} (\tan_A(2\sqrt{\wp\lambda}\zeta) \pm \sqrt{uv} \sec_A(2\sqrt{\wp\lambda}\zeta)), \quad (19)$$

$$\mathfrak{F}_{14}(\zeta) = \sqrt{\frac{\wp}{\lambda}} (-\cot_A(2\sqrt{\wp\lambda}\zeta) \pm \sqrt{uv} \csc_A(2\sqrt{\wp\lambda}\zeta)), \quad (20)$$

$$\mathfrak{F}_{15}(\zeta) = \frac{1}{2}\sqrt{\frac{\wp}{\lambda}}(\tan_A(\frac{\sqrt{\wp\lambda}}{2}\zeta) - \cot_A(\frac{\sqrt{\wp\lambda}}{2}\zeta)). \quad (21)$$

(Case 4): When $\wp\lambda < 0$ and $\mathfrak{B} = 0$,

$$\mathfrak{F}_{16}(\zeta) = -\sqrt{-\frac{\wp}{\lambda}} \tanh_A(\sqrt{-\wp\lambda}\zeta), \quad (22)$$

$$\mathfrak{F}_{17}(\zeta) = -\sqrt{-\frac{\wp}{\lambda}} \coth_A(\sqrt{-\wp\lambda}\zeta), \quad (23)$$

$$\mathfrak{F}_{18}(\zeta) = \sqrt{-\frac{\wp}{\lambda}}(-\tanh_A(2\sqrt{-\wp\lambda}\zeta) \pm \iota\sqrt{uv} \operatorname{sech}_A(2\sqrt{-\wp\lambda}\zeta)), \quad (24)$$

$$\mathfrak{F}_{19}(\zeta) = \sqrt{-\frac{\wp}{\lambda}}(-\coth_A(2\sqrt{-\wp\lambda}\zeta) \pm \sqrt{uv} \operatorname{csch}_A(2\sqrt{-\wp\lambda}\zeta)), \quad (25)$$

$$\mathfrak{F}_{20}(\zeta) = -\frac{1}{2}\sqrt{-\frac{\wp}{\lambda}}(\tanh_A(\frac{\sqrt{-\wp\lambda}}{2}\zeta) + \coth_A(\frac{\sqrt{-\wp\lambda}}{2}\zeta)). \quad (26)$$

(Case 5): When $\mathfrak{B} = 0$ and $\wp = \lambda$,

$$\mathfrak{F}_{21}(\zeta) = \tan_A(\wp\zeta), \quad (27)$$

$$\mathfrak{F}_{22}(\zeta) = -\cot_A(\wp\zeta), \quad (28)$$

$$\mathfrak{F}_{23}(\zeta) = \tan_A(2\wp\zeta) \pm \sqrt{uv} \sec_A(2\wp\zeta), \quad (29)$$

$$\mathfrak{F}_{24}(\zeta) = -\cot_A(2\wp\zeta) \pm \sqrt{uv} \csc_A(2\wp\zeta), \quad (30)$$

$$\mathfrak{F}_{25}(\zeta) = \frac{1}{2}(\tan_A(\frac{\wp}{2}\zeta) - \cot_A(\frac{\wp}{2}\zeta)). \quad (31)$$

(Case 6): When $\mathfrak{B} = 0$ and $\wp = -\lambda$,

$$\mathfrak{F}_{26}(\zeta) = -\tanh_A(\wp\zeta), \quad (32)$$

$$\mathfrak{F}_{27}(\zeta) = -\coth_A(\wp\zeta), \quad (33)$$

$$\mathfrak{F}_{28}(\zeta) = -\tanh_A(2\wp\zeta) \pm \iota\sqrt{uv} \operatorname{sech}_A(2\wp\zeta), \quad (34)$$

$$\mathfrak{F}_{29}(\zeta) = -\cot_A(2\wp\zeta) \pm \sqrt{uv} \operatorname{csch}_A(2\wp\zeta), \quad (35)$$

$$\mathfrak{F}_{30}(\zeta) = -\frac{1}{2}(\tanh_A(\frac{\wp}{2}\zeta) + \coth_A(\frac{\wp}{2}\zeta)). \quad (36)$$

(Case 7): When $\mathfrak{B}^2 = 4\wp\lambda$,

$$\mathfrak{F}_{31}(\zeta) = \frac{-2\wp(\mathfrak{B}\zeta \ln(A) + 2)}{\mathfrak{B}^2\zeta \ln(A)}. \quad (37)$$

(Case 8): When $\mathfrak{B} = p$, $\wp = pq$, ($q \neq 0$) and $\lambda = 0$,

$$\mathfrak{F}_{32}(\zeta) = A^{p\zeta} - q. \quad (38)$$

(Case 9): When $\mathfrak{B} = \lambda = 0$,

$$\mathfrak{F}_{33}(\zeta) = \wp \zeta \ln(A). \quad (39)$$

(Case 10): When $\wp = \mathfrak{B} = 0$,

$$\mathfrak{F}_{34}(\zeta) = \frac{-1}{\lambda \zeta \ln(A)}. \quad (40)$$

(Case 11): When $\wp = 0$, and $\mathfrak{B} \neq 0$,

$$\mathfrak{F}_{35}(\zeta) = -\frac{u\mathfrak{B}}{\lambda(\cosh_A(\mathfrak{B}\zeta) - \sinh_A(\mathfrak{B}\zeta) + u)}, \quad (41)$$

$$\mathfrak{F}_{36}(\zeta) = -\frac{\mathfrak{B}(\sinh_A(\mathfrak{B}\zeta) + \cosh_A(\mathfrak{B}\zeta))}{\lambda(\sinh_A(\mathfrak{B}\zeta) + \cosh_A(\mathfrak{B}\zeta) + v)}. \quad (42)$$

(Case 12): When $\mathfrak{B} = p$, $\lambda = pq$, ($q \neq 0$) and $\wp = 0$,

$$\mathfrak{F}_{37}(\zeta) = -\frac{uA^{p\zeta}}{u - qvA^{p\zeta}}. \quad (43)$$

$$\begin{aligned} \sinh_A(\zeta) &= \frac{uA^\zeta - vA^{-\zeta}}{2}, & \cosh_A(\zeta) &= \frac{uA^\zeta + vA^{-\zeta}}{2}, \\ \tanh_A(\zeta) &= \frac{uA^\zeta - vA^{-\zeta}}{uA^\zeta + vA^{-\zeta}}, & \coth_A(\zeta) &= \frac{uA^\zeta + vA^{-\zeta}}{uA^\zeta - vA^{-\zeta}}, \\ \operatorname{sech}_A(\zeta) &= \frac{2}{uA^\zeta + vA^{-\zeta}}, & \operatorname{csch}_A(\zeta) &= \frac{2}{uA^\zeta - vA^{-\zeta}}, \\ \sin_A(\zeta) &= \frac{uA^{\iota\zeta} - vA^{-\iota\zeta}}{2\iota}, & \cos_A(\zeta) &= \frac{uA^{\iota\zeta} + vA^{-\iota\zeta}}{2}, \\ \tan_A(\zeta) &= -\iota \frac{uA^{\iota\zeta} - vA^{-\iota\zeta}}{uA^{\iota\zeta} + vA^{-\iota\zeta}}, & \cot_A(\zeta) &= \iota \frac{uA^{\iota\zeta} + vA^{-\iota\zeta}}{uA^{\iota\zeta} - vA^{-\iota\zeta}}, \\ \sec_A(\zeta) &= \frac{2}{uA^\zeta + vA^{-\zeta}}, & \csc_A(\zeta) &= \frac{2\iota}{uA^\zeta - vA^{-\zeta}}. \end{aligned} \quad (44)$$

The deformation arbitrary constants parameters u and v are greater than zero.

2.2 Exact solutions for Clannish Random Walkers Parabolic equation

We use the following travelling wave To obtain the solutions of Eq. (1), transformation:

$$\mathfrak{U}(x, t) = \mathfrak{U}(\zeta) \text{ where } \zeta = x - \frac{c}{\Omega} \left(t + \frac{1}{\lambda(\Omega)} \right)^\Omega, \quad (45)$$

where c is constant. On the contrivance of Eq. (45) to the Eq. (1), we reveal that:

$$2(j - c)N + kN^2 + 2lN' = 0. \quad (46)$$

According to the homogeneous balancing principle of Eq. (46) gives $M = 1$.

$$\mathfrak{U}(x, t) = a_0 + a_1 \mathfrak{F}(\zeta), \quad (47)$$

where,

$$\mathfrak{F}'(\zeta) = \ln(A)(\wp + \mathfrak{B}\mathfrak{F}(\zeta) + \lambda\mathfrak{F}^2(\zeta)), \quad A \neq 0, 1, \quad (48)$$

The prognosticate solution Eq. (47) is plugging in Eq. (46) and equating the coefficient of distinct power of $\mathfrak{F}(\zeta)$ prevailed the algebraic system of equations:

$$\begin{aligned} (\mathfrak{F}(\zeta))^0 : & 2ja_0 - 2ca_0 + ka_0^2 + 2la_1 \ln(A)\wp, \\ (\mathfrak{F}(\zeta))^1 : & 2ja_1 - 2ca_1 + 2ka_0a_1 + 2la_1 \ln(A)\mathfrak{B}, \\ (\mathfrak{F}(\zeta))^2 : & ka_1^2 + 2la_1 \ln(A)\lambda. \end{aligned} \quad (49)$$

system of Eq. (49) is solved with the help of the modern software **Mathematica** to get the required parameters,

$$a_0 = \frac{l \ln(A)}{k}(\mathfrak{B} \pm \sqrt{S}), \quad a_1 = \frac{2l \ln(A)}{k}\lambda, \quad c = j + \ln(A)(\mathfrak{B}l - \mathfrak{B} \pm l\sqrt{S}). \quad (50)$$

$$\mathfrak{U}(x, t) = \Omega \mathfrak{B} \pm \Omega \sqrt{S} + 2\Omega \lambda [\mathfrak{F}_i(\zeta)]. \quad (51)$$

Where, $\Omega = \frac{\ln(A)l}{k}$ and $S = \mathfrak{B}^2 - 4\wp\lambda$.

Exact solutions for Eq. (1) can be obtained by plugging the Eq. (50) into Eq. (47), which is given below,

(Case 1): When $\mathfrak{B}^2 - 4\wp\lambda < 0$ and $\lambda \neq 0$,

$$\mathfrak{U}_1(x, t) = \Omega \sqrt{S} (1 + \iota \tan_A(\frac{\sqrt{-S}}{2}\zeta)), \quad (52)$$

$$\mathfrak{U}_2(x, t) = \Omega \sqrt{S} (1 - \iota \cot_A(\frac{\sqrt{S}}{2}\zeta)), \quad (53)$$

$$\mathfrak{U}_3(x, t) = \Omega \sqrt{S} (1 + \iota (\tan_A(\sqrt{-S}\zeta) \pm \sqrt{uv} \sec_A(\sqrt{-S}\zeta))), \quad (54)$$

$$\mathfrak{U}_4(x, t) = \Omega \sqrt{S} (1 + \iota (\cot_A(\sqrt{-S}\zeta) \pm \sqrt{uv} \csc_A(\sqrt{-S}\zeta))), \quad (55)$$

$$\mathfrak{U}_5(x, t) = \Omega \sqrt{S} (1 + \frac{\iota}{2} (\tan_A(\frac{\sqrt{-S}}{4}\zeta) - \cot_A(\frac{\sqrt{-S}}{4}\zeta))). \quad (56)$$

(Case 2): When $\mathfrak{B}^2 - 4\wp\lambda > 0$ and $\lambda \neq 0$,

$$\mathfrak{U}_6(x, t) = \Omega\sqrt{S}(1 - \tan_A(\frac{\sqrt{S}}{2}\zeta)), \quad (57)$$

$$\mathfrak{U}_7(x, t) = \Omega\sqrt{S}(1 - \cot_A(\frac{\sqrt{S}}{2}\zeta)), \quad (58)$$

$$\mathfrak{U}_8(x, t) = \Omega\sqrt{S}(1 - \tan_A(\sqrt{S}\zeta) \pm \iota\sqrt{uv}\sec_A(\sqrt{S}\zeta)), \quad (59)$$

$$\mathfrak{U}_9(x, t) = \Omega\sqrt{S}(1 - \cot_A(\sqrt{S}\zeta) \pm \sqrt{uv}\csc_A(\sqrt{S}\zeta)), \quad (60)$$

$$\mathfrak{U}_{10}(x, t) = \Omega\sqrt{S}(1 + \frac{1}{2}(\tan_A(\frac{\sqrt{S}}{4}\zeta) + \cot_A(\frac{\sqrt{S}}{4}\zeta))). \quad (61)$$

(Case 3): When $\wp\lambda > 0$ and $\mathfrak{B} = 0$,

$$\mathfrak{U}_{11}(x, t) = 2\Omega\sqrt{\lambda}(\sqrt{\wp}\tan_A(\sqrt{\wp\lambda}\zeta + \sqrt{\wp}\iota), \quad (62)$$

$$\mathfrak{U}_{12}(x, t) = 2\Omega\sqrt{\lambda}(-\sqrt{\wp}\cot_A(\sqrt{\wp\lambda}\zeta + \sqrt{\wp}\iota), \quad (63)$$

$$\mathfrak{U}_{13}(x, t) = 2\Omega\sqrt{\lambda}(\sqrt{\wp}(\tan_A(2\sqrt{\wp\lambda}\zeta) \pm \sqrt{uv}\sec_A(2\sqrt{\wp\lambda}\zeta)) + \sqrt{\wp}\iota), \quad (64)$$

$$\mathfrak{U}_{15}(x, t) = 2\Omega\sqrt{\lambda}(\frac{\sqrt{\wp}}{2}(\tan_A(\frac{\sqrt{\wp\lambda}}{2}\zeta - \cot_A(\frac{\sqrt{\wp\lambda}}{2}\zeta)) + \sqrt{\wp}\iota). \quad (65)$$

(Case 4): When $\wp\lambda < 0$ and $\mathfrak{B} = 0$,

$$\mathfrak{U}_{16}(x, t) = 2\Omega\sqrt{\lambda}(-\sqrt{-\wp}\tanh_A(\sqrt{-\wp\lambda}\zeta) + \sqrt{\wp}\iota), \quad (66)$$

$$\mathfrak{U}_{17}(x, t) = 2\Omega\sqrt{\lambda}(-\sqrt{-\wp}\coth_A(\sqrt{-\wp\lambda}\zeta) + \sqrt{\wp}\iota), \quad (67)$$

$$\mathfrak{U}_{18}(x, t) = 2\Omega\sqrt{\lambda}(\sqrt{-\wp}(-\tanh_A(2\sqrt{-\wp\lambda}\zeta) \pm \iota\sqrt{uv}\operatorname{sech}_A(2\sqrt{-\wp\lambda}\zeta)) + \sqrt{\wp}\iota), \quad (68)$$

$$\mathfrak{U}_{19}(x, t) = 2\Omega\sqrt{\lambda}(\sqrt{-\wp}(-\coth_A(2\sqrt{-\wp\lambda}\zeta \pm \sqrt{uv}\operatorname{csch}_A(2\sqrt{-\wp\lambda}\zeta)) + \sqrt{\wp}\iota), \quad (69)$$

$$\mathfrak{U}_{20}(x, t) = 2\Omega\sqrt{\lambda}(-\frac{\sqrt{-\wp}}{2}(\tanh_A(\frac{\sqrt{-\wp\lambda}}{2}\zeta) + \coth(\frac{\sqrt{-\wp\lambda}}{+}\sqrt{\wp}\iota). \quad (70)$$

(Case 5): When $\mathfrak{B} = 0$ and $\wp = \lambda$,

$$\mathfrak{U}_{21}(x, t) = 2\Omega\wp(\tan_A(\wp\zeta) + \iota), \quad (71)$$

$$\mathfrak{U}_{22}(x, t) = 2\Omega\wp(-\cot_A(\wp\zeta) + \iota), \quad (72)$$

$$\mathfrak{U}_{23}(x, t) = 2\Omega\wp(\tan_A(2\wp\zeta) \pm \sqrt{uv}\sec_A(2\wp\zeta) + \iota), \quad (73)$$

$$\mathfrak{U}_{24}(x, t) = 2\Omega\wp(-\cot_A(2\wp\zeta) \pm \sqrt{uv}\csc_A(2\wp\zeta) + \iota), \quad (74)$$

$$\mathfrak{U}_{25}(x, t) = 2\Omega\wp\left(\frac{1}{2}\tan_A\left(\frac{\wp}{2}\zeta\right) - \cot_A\left(\frac{\wp}{2}\zeta\right) + \iota\right). \quad (75)$$

(Case 6): When $\mathfrak{B} = 0$ and $\lambda = -\wp$,

$$\mathfrak{U}_{26}(x, t) = 2\Omega\wp(1 + \tanh_A(\wp\zeta)), \quad (76)$$

$$\mathfrak{U}_{27}(x, t) = 2\Omega\wp(1 + \coth_A(\wp\zeta)), \quad (77)$$

$$\mathfrak{U}_{28}(x, t) = 2\Omega\wp(1 + \tanh_A(2\wp\zeta) \mp \iota\sqrt{uv}\sec_A(2\wp\zeta)), \quad (78)$$

$$\mathfrak{U}_{29}(x, t) = 2\Omega\wp(1 + \cot_A(2\wp\zeta) \mp \sqrt{uv}\operatorname{csch}_A(2\wp\zeta)), \quad (79)$$

$$\mathfrak{U}_{30}(x, t) = 2\Omega\wp\left(1 + \frac{1}{2}\left(\tanh_A\left(\frac{\wp}{2}\zeta\right) - \cot_A\left(\frac{\wp}{2}\zeta\right)\right)\right). \quad (80)$$

(Case 7): When $\mathfrak{B}^2 = 4\wp\lambda$,

$$\mathfrak{U}_{31}(x, t) = \Omega\mathfrak{B} + 2\Omega\lambda\left(-\frac{2\wp(\mathfrak{B}\zeta\ln(A) + 2)}{\mathfrak{B}^2\zeta\ln(A)}\right). \quad (81)$$

(Case 8): When $\mathfrak{B} = p$, $\wp = pq$, ($q \neq 0$) and $\lambda = 0$,

$$\mathfrak{U}_{32}(x, t) = 2\Omega p. \quad (82)$$

(Case 9): When $\mathfrak{B} = \lambda = 0$,

$$\mathfrak{U}_{33}(x, t) = 0. \quad (83)$$

(Case 10): When $\wp = \mathfrak{B} = 0$,

$$\mathfrak{U}_{34}(x, t) = 2\Omega\left(\frac{-1}{\zeta\ln(A)}\right). \quad (84)$$

(Case 11): When $\wp = 0$, and $\mathfrak{B} \neq 0$,

$$\mathfrak{U}_{35}(x, t) = 2\Omega\left(\frac{-u\wp}{\cosh_A(\wp\zeta) - \sinh_A(\wp\zeta) + u} + \iota\sqrt{\wp\lambda}\right), \quad (85)$$

$$\mathfrak{U}_{36}(x, t) = 2\Omega\left(\frac{-\wp(\sinh_A(\wp\zeta) + \cosh_A(\wp\zeta))}{\sinh_A(\wp\zeta) + \cosh_A(\wp\zeta) + v} + \iota\sqrt{\wp\lambda}\right). \quad (86)$$

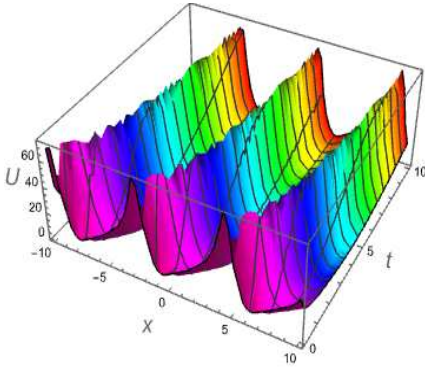
(Case 12): When $\mathfrak{B} = p$, $\lambda = pq$, ($q \neq 0$) and $\wp = 0$,

$$\mathfrak{U}_{37}(x, t) = 2\Omega p\left(\frac{-uA^p\zeta}{u - qvA^p\zeta} + \iota\sqrt{q}\right). \quad (87)$$

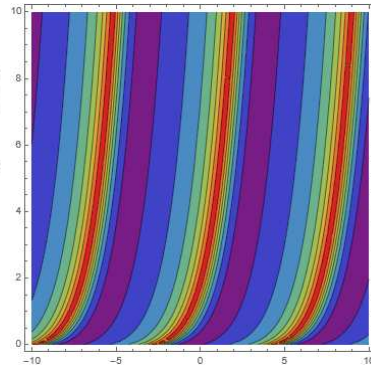
3 Graphical Discussion

This section explains numerical and physical simulations of some obtained solutions to the successive non-linear evolution equations of the time-fractional Clannish Random Walkers Parabolic equation by selecting appropriate values for the arbitrary parameters. A modern software program Wolfram Mathematica is utilized to plot graphs for better presentation.

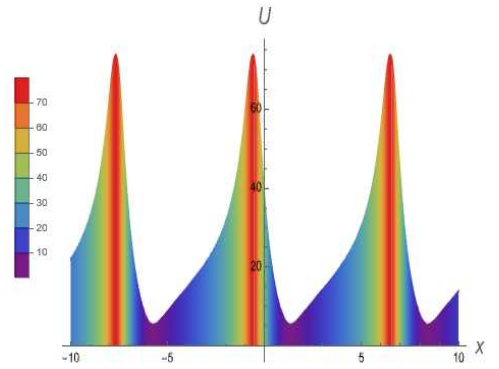
In Figure (1), graphs for the soliton wave number (Ω) of the obtained solution $\mathfrak{U}_1(x, t)$, at the parametric values $\wp=0.6$, $\mathfrak{B}=0.2$, $\lambda=0.7$, $A = 2$, $l = 5$, $k=0.4$, $u=3$, $v=2$ and $c=1$, for different values of wave numbers, in the form of 3-D, 2-D and contour. In Fig (1a) at $\Omega=0.1$, continuous bright solitonic behaviour is observed in a 3-D profile, by increasing the value of wave number we found the same behaviour in fig (1d) and (1g) and for more visualization contour was plotted and found the bright soliton in (1b), (1e) and (1h), and 2-D shows the bright periodic soliton in (1c), (1f) and (1i) by increasing values of wave speed. In Figure (2), graphs for the soliton velocity of the obtained solutions $\mathfrak{U}_{25}(x, t)$, at the parametric values $\wp= 5$, $\mathfrak{B}= 0$, $\lambda= 5$, $A = 3$, $l = 5$, $k = 0.4$, $u = 2.9$, $v = 6$, $c = 0.9$, for different values of wave number, in the form of 3-D, 2-D and contour, multiple bright solitonic behaviours are observed in 3-D, bright solitons in contour and multiple bright singular solitons can be seen in 2-D for the same values of wave number as we took in Fig (1). In Figure 3, graphs for $\mathfrak{U}_{28}(x, t)$ for parametric values $\wp=5$, $\mathfrak{B}=0$, $\lambda=-5$, $A = 2$, $l = 5$, $k = 0.4$, $u = 3$, $v = 2$, $c = 1$ at wave speed $\Omega=0.1$, shows flat anti-kink structure in 3-D, contour and 2-D getting bright by increasing the value of soliton eave number. Moreover, it is expected that future studies on non-linear wave difficulties in applied sciences will be helped by the results presented and these explanations of our novel work.



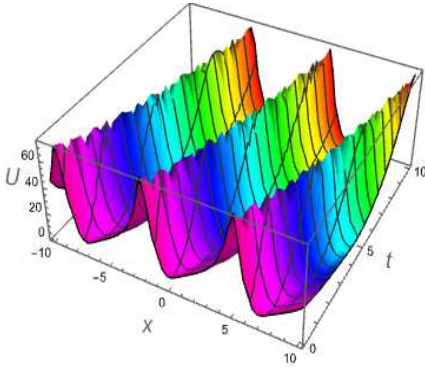
(a) 3-D graphical profile of soliton wave number at $\Omega = 0.1$



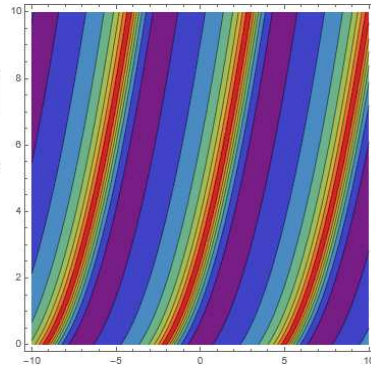
(b) Contour graphical profile of soliton wave number at $\Omega = 0.1$



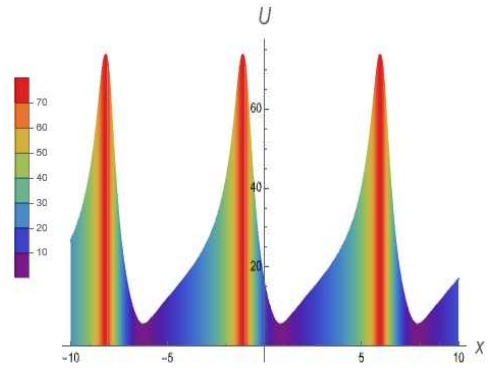
(c) 3-D graphical profile of soliton wave number at $\Omega = 0.1$



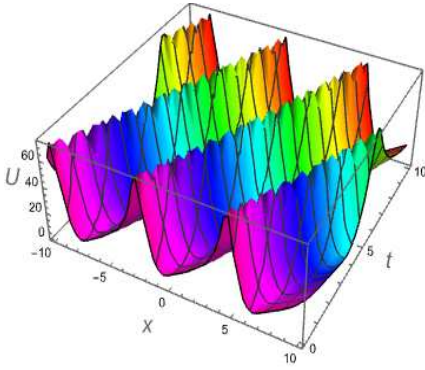
(d) 3-D graphical profile of soliton wave number at $\Omega = 0.5$



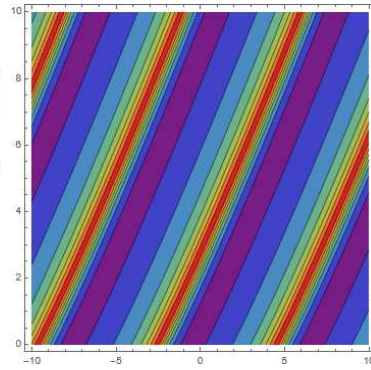
(e) Contour graphical profile of soliton wave number at $\Omega = 0.5$



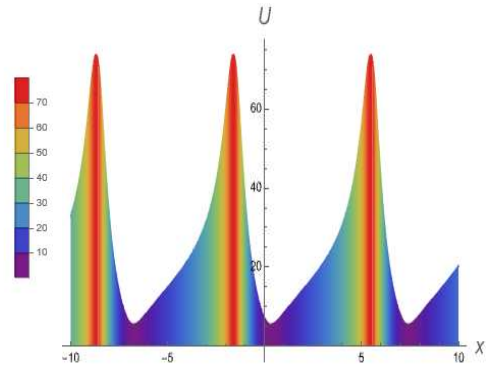
(f) 3-D graphical profile of soliton wave number at $\Omega = 0.5$



(g) 3-D graphical profile of soliton wave number at $\Omega = 0.9$



(h) Contour graphical profile of soliton wave number at $\Omega = 0.9$



(i) 3-D graphical profile of soliton wave number at $\Omega = 0.9$

Figure 1: Visualised impact of wave number in 3-D, 2-D and contour for solution $\mathfrak{U}_1(x, t)$

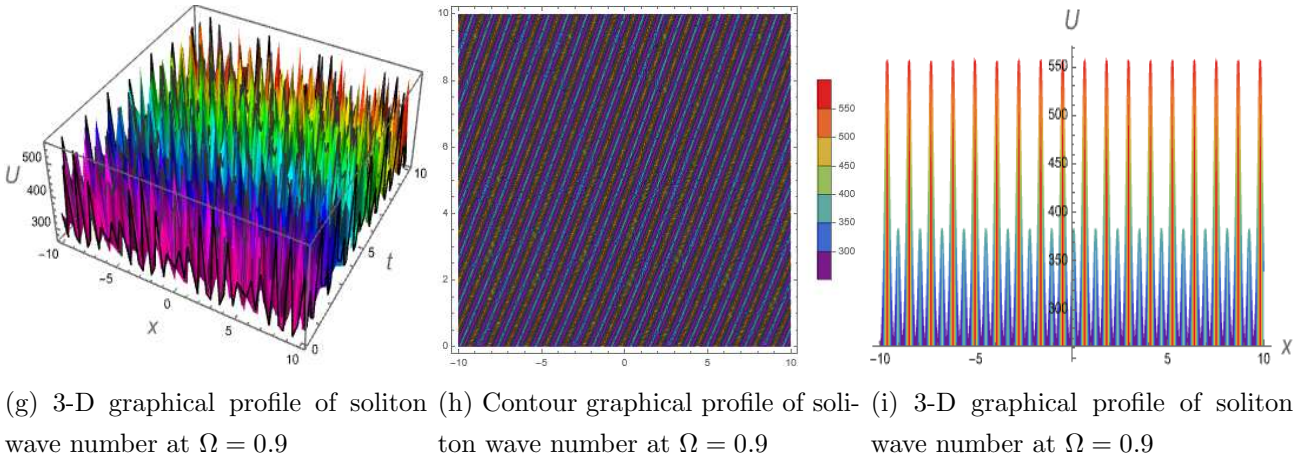
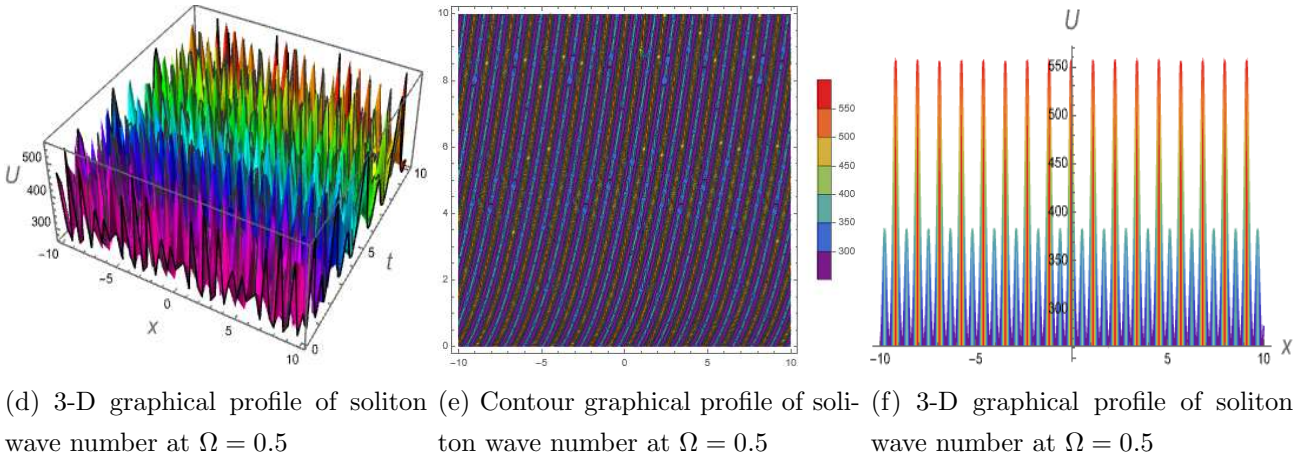
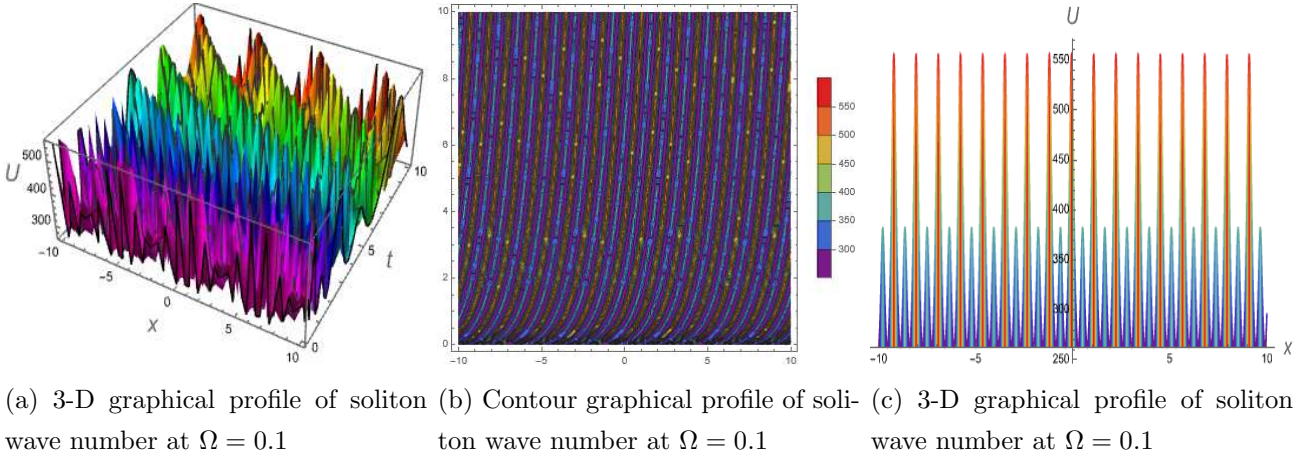
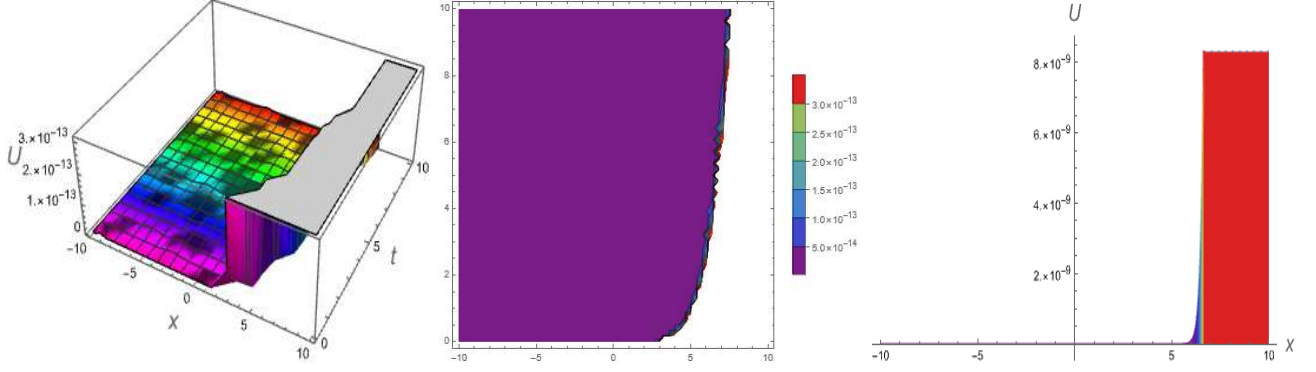
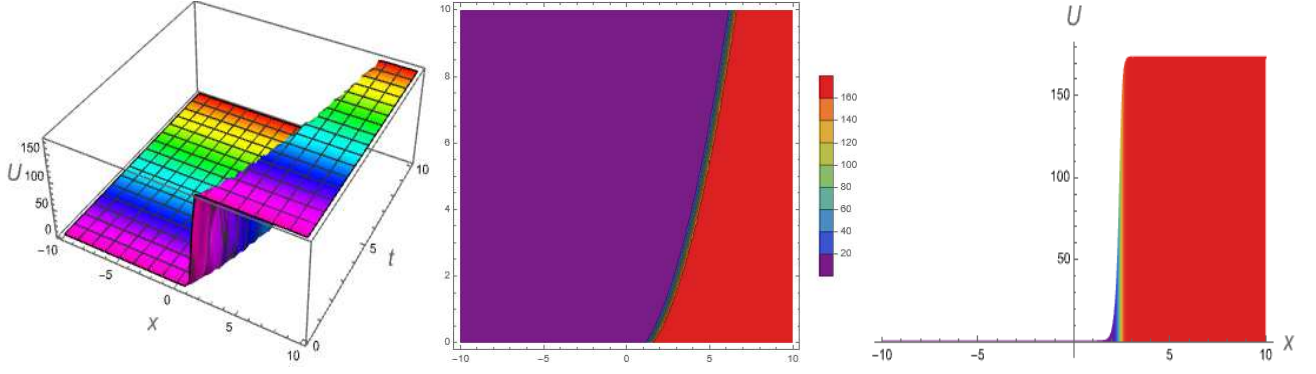


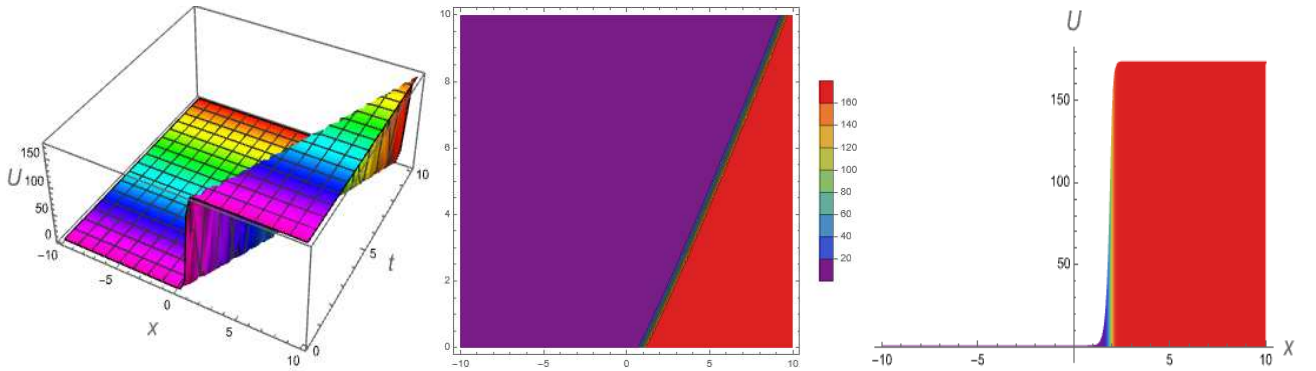
Figure 2: Visualised impact of wave number in 3-D, 2-D and contour for solution $\mathcal{U}_{25}(x, t)$



(a) 3-D graphical profile of soliton wave number at $\Omega = 0.1$ (b) Contour graphical profile of soliton wave number at $\Omega = 0.1$ (c) 3-D graphical profile of soliton wave number at $\Omega = 0.1$



(d) 3-D graphical profile of soliton wave number at $\Omega = 0.5$ (e) Contour graphical profile of soliton wave number at $\Omega = 0.5$ (f) 3-D graphical profile of soliton wave number at $\Omega = 0.5$



(g) 3-D graphical profile of soliton wave number at $\Omega = 0.9$ (h) Contour graphical profile of soliton wave number at $\Omega = 0.9$ (i) 3-D graphical profile of soliton wave number at $\Omega = 0.9$

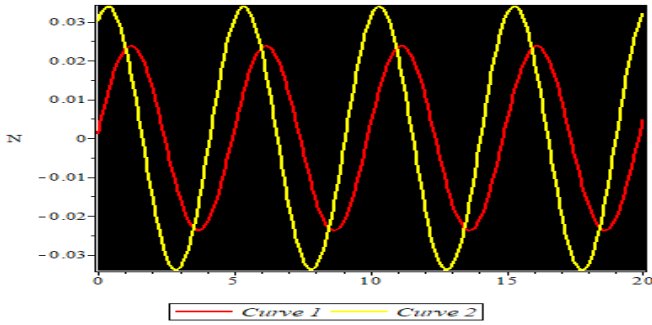
Figure 3: Visualised impact of wave number in 3-D, 2-D and Contour for soliton solution $\mathfrak{U}_{28}(x, t)$

3.1 The sensitivity assessment

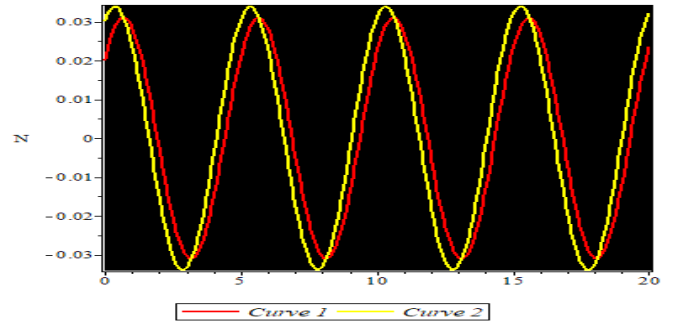
The plane dynamic system can be used to contribute to the sensitivity of the time-fractional Clannish Random Walkers Parabolic equation, the dynamic system of Eq. (46) is as follows:

$$\left\{ N' = \frac{-2(j-c)N}{2l} - \frac{kN^2}{2l} = 0. \right. \quad (88)$$

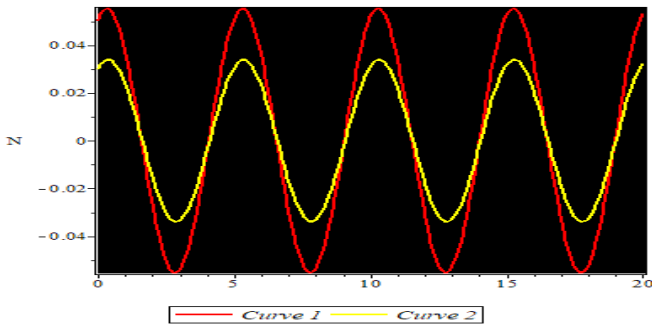
This section discusses the sensitive behaviour of the considered model with parametric values $c = 1.2$, $k = 0.9$, $j = 1.5$ and $l = 0.5$, after transforming it into a dynamical system (88). The sensitivity of the dynamical system depends on the initial guess. By choosing a suitable initial guess sensitivity of the system can be controlled. If a minor adjustment in the initial conditions results in a slight change in the system, then the system is less sensitive. We will establish the physical properties of the considered model and discuss the effects of the frequency and perturbations force. As a result, by using different initial conditions in the segment, we recognize the sensitivity of the solution of the dynamical system.



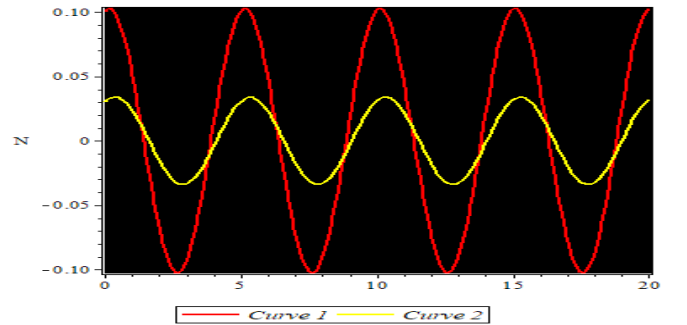
(a) Sensitive analysis for the curve 1 and curve 2 at (0.001, 0.03) and (0.03, 0.02) respectively.



(b) Sensitive analysis for the curve 1 and curve 2 at (0.02, 0.03) and (0.03, 0.02) respectively.



(c) Sensitive analysis for the curve the 1 and curve 2 at (0.05, 0.03) and (0.03, 0.02) respectively.



(d) Sensitive analysis for the curve 1 and curve 2, at (0.1, 0.03) and (0.03, 0.02) respectively.

Figure 4: Sensitivity visualization at distinct initial conditions

In Fig. (4), the main goal of the present investigation is to accurately assess the output disruption caused by input changes. The findings of the analysis show different parametric values to illustrate how small changes in input can result in large variances in the result. In Fig. (7a), it can be illustrated that a slight change in the initial condition affects the solutions that contribute to the disorderly behaviour of the curve. The same experiment is replicated in Figs. (6b), (6c) and (6d), by retaining the values of the parameters and making a larger change in the initial conditions, and the same effects are observed, as a result, in this case, the system is sensitive.

4 Conclusions

In this article, the new-extended direct algebraic technique is used to identify the new exact travelling wave solutions of the time-fractional (CRWP) model successfully. 3-D, 2-D and contour presentation of some newly derived solutions by giving specific values to the parameters under the constraint conditions with the help of **Mathematica** is illustrated. The singularity of water waves can be controlled for larger values of wave number. The sensitive analysis is observed in an obtained system for the initial guess and other parametric values respectively. It is important to highlight that using our analytical modified mathematical methodology has resulted in the finding of these new exact solitary wave solutions to the time-fractional (CRWP) equation that has not before been reported in the literature. Finally, the explored solutions have the potential to be used to solve a variety of non-linear (PDEs) that are evident in many significant non-linear scientific phenomena in engineering and physics.

Declarations

Ethics approval and consent to participate

Not applicable

Consent for publication

Not applicable

Competing interests

The authors state that they do not have any competing interests.

Authors' contributions

Problem formulation, formal analysis S.Z.M and H.A; Investigation, Methodology W.A.F and M.I.A, Supervision, resources; S.Z.M, M.I.A Validation, software and graphical discussion; H.A and W.A.F. editing and Review; all authors approved the final version for submission.

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Availability of material and data

As no data sets were collected or analyzed during the present investigation, data sharing is not applicable to this paper.

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